ON SOME EPICUREAN AND LUCRETIAN ARGUMENTS FOR THE INFINITY OF THE UNIVERSE

As is well known, Epicurus and his followers held that the universe was infinite and that its two primary components, void and atoms, were each infinite. The void was infinite in extension, the atoms were infinite in number and their total was infinite also in extension. The chief Epicurean proofs of these infinities are found in Epicurus, Ad Herod. 41–2, and in Lucretius 1. 951–1020. As far as I can see, both the commentators to these works and writers on Epicurean physics in general have neglected to take into account some material pertinent to these proofs, material found in Aristotle and especially in his commentators Alexander of Aphrodisias, Themistius, Simplicius, and Philoponus. In this article I wish to compare this neglected information with the proofs of infinity found in Epicurus and Lucretius and to discuss their authorship.

In Physics 203 b 15–30 Aristotle gives five anonymous arguments which according to him have induced belief in infinity. The fourth argument asserts that infinity exists because anything finite is always bounded by something else: $\hat{\eta}$ πίστις... ἔτι τῷ τὸ πεπερασμένον ἀεὶ πρός τι περαίνειν, ὥστε ἀνάγκη μηδὲν εἶναι πέρας, εἶ ἀεὶ περαίνειν ἀνάγκη ἔτερον πρὸς ἔτερον (203 b 20–2).

This passage in Aristotle is commented on at greater or lesser length by Alexander of Aphrodisias, Themistius, Simplicius, and Philoponus. In Alexander, Themistius, and Simplicius we learn that Epicurus or the Epicureans were using a somewhat expanded version of this fourth proof in Aristotle to prove the infinity of the universe $(\tau \hat{\sigma} \pi \hat{a} \nu)$. The fullest version of this Epicurean proof is found in Simplicius (above, n. 5, 466. 31–467. 4): $\tau \hat{\epsilon} \tau a \rho \tau o \nu$ (sc. the fourth proof) $\delta \hat{\epsilon} \kappa a \hat{\iota} \delta \nu \sigma a \nu \tau \hat{\iota} \beta \lambda \epsilon \pi \tau o \nu \tau \hat{\iota}$ φαίνεσθαι πᾶν τὸ πεπερασμένον ἀεὶ πρός τι περαῖνον. εἰ γὰρ πᾶν τὸ πεπερασμένον

- ¹ For infinity of universe, void, and matter see Epicurus, Ad Herod. 41–2. The spatial infinity of matter results from the statement of Epicurus here: If the void were finite, the infinity of bodies would have no place to be. That the extension of the infinite number of atoms must be infinite, results also from Ad Herod. 57. Here Epicurus states that if any particle has size (as atoms have), an infinite number of them would be of infinite extension.
- ² Some notice of pertinent passages in Aristotle and in Simplicius was taken by P. Gassendi, Opera Omnia 1 (Lyons, 1658), 188. A mere reference to Gassendi's parallels was given by A. J. Reisacker, Quaestiones Lucretianae (Bonn, 1847), 25 n. 1. Reisacker's note was in turn referred to by F. Hildebrandt, T. Lucretii de primordiis doctrina (Magdeburg, 1864), 28 n. 1. The most specific use of any of this material was made by Th. Bindseil, Nonnulla ad Lucretii de omnis infinitate doctrinam (Berlin, 1870), 7–8, who noticed that Lucretius' proof of infinity in 1. 1008–13 was related to the fourth proof of infinity in Aristotle, Physics 203 b 20–3.
- ³ Ivo Bruns (ed.), 'Alexandri Aphrodisiensis praeter commentaria scripta minora', Quaestiones 3. 12, Supplementum Aristotelicum II. 2 (Berlin, 1892), 101–7. Of this passage p. 104. 20–3 is in Usener, Epicurea, no. 297; p. 104. 20–6 ibid. no. 301.
- ⁴ H. Schenkl (ed.), 'Themistii in Aristotelis Physica paraphrasis', Commentaria in Aristotelem graeca v (Berlin, 1900), 81. 28–30, 94. 6–9, and 99. 15–100. 23. 100. 6–11 is also in Us., no. 298.
- ⁵ H. Diels (ed.) 'Simplicii in Aristotelis Physicorum libros quattuor priores commentaria', *Commentaria in Aristotelem graeca* IX (Berlin, 1882), 466. 31–467. 4, 500. 8–11 and 516. 3–38. Us., no. 297 has pp. 466. 31–467. 4.
- ⁶ G. Vitelli (ed.), 'Ioannis Philoponi in Aristotelis Physicorum libros tres priores commentaria', *Commentaria in Aristotelem graeca* xvi (Berlin, 1887), 405. 7–14, 473. 24–474. 6 and 494. 2–24.

πρὸς ἄλλο τι ἔξωθεν αὐτοῦ ὂν περαίνει, τὸ ἔξωθεν ἐκεῖνο πρὸς ὅ περαίνει, ἢ ἄπειρόν ἐστιν ἢ πεπερασμένον. καὶ εἰ μὲν ἄπειρον, ἔχομεν αὐτόθεν ὅτι ἔστι τὸ ἄπειρον· εἰ δὲ πεπερασμένον οἶον ἡ γῆ, καὶ αὐτὸ πρὸς ἄλλο περαίνει, καὶ τοῦτο ἐπ' ἄπειρον. εἰ δὲ ἐπ' ἄπειρον, ἔστι τὸ ἄπειρον. οὐδὲν γὰρ τελευταῖον ληφθήσεται πέρας, εἴπερ καὶ τοῦτο πρὸς ἄλλο τι περαίνει. ''τούτῳ δὲ [μάλιστα] τῷ λόγῳ, ὡς μάλιστα, ὡς φησιν ὁ ᾿Αλέξανδρος, τοἱ περὶ Ἐπίκουρον πιστεύοντες ἄπειρον ἔλεγον εἶναι τὸ πᾶν, διότι πᾶν τὸ πεπερασμένον παρά τι περαινόμενον ἔξωθέν τι ἔχει.'' ὁ δὲ ᾿Αριστοτέλης ὡς ἀρχαιοτέρου μέμνηται τοῦ λόγου.8

As we can see, the second part of the Epicurean proof also contains the observation, already implicit in its first part, that infinity would also result if a finite quantum were succeeded by an infinite one. This additional proof is also found in Lucretius 1. 1008–13 but not in the extant writings of Epicurus. A similar notion is attributed to Eudemus, the student of Aristotle, who observes that time consists of two segments, the present and the infinite past: $\delta o \kappa \epsilon \hat{\iota} \gamma \dot{a} \rho \ \delta \ \pi a \rho \epsilon \lambda \eta \theta \upsilon \theta \dot{\omega} s \ \chi \rho \acute{o} vos \ \tilde{a} \pi \epsilon \iota \rho os \ \epsilon \tilde{\iota} v a \iota \pi \epsilon \rho a \iota v \alpha \nu \ \pi \rho \dot{o} s \ \tau \dot{o} \nu \ \pi a \rho \acute{o} \nu \tau a (Wehrli², VIII, Fr. 41).$

A comparison of the versions of this Epicurean proof in the commentators yields these main points:

- (1) Anything finite is limited by something external to it.
- (2) Finite quanta keep limiting each other to infinity.
- (3) Infinity also results if a finite quantum (or quanta) is succeeded by an infinite one.

Point two is implied by point one. Point three seems self-evident.

Although Simplicius on the authority of Alexander states that to prove the infinity of the universe the Epicureans placed especial confidence in this proof, it is not found in the extant writings of Epicurus. It is in evidence, however, in Lucretius 1. 1008–13, an indication that the *Letter to Herodotus* of Epicurus was not his source here.

The similarity of the proof in 1. 1008-13 to the new Epicurean proof is patent. Both proofs profess to prove the existence of infinity and both offer the same arguments. Point one of the Epicurean proof appears in lines 1009-10: natura...corpus...finiri corpore cogit. Point two is found in 1011: alternis infinita omnia (sc. redduntur). To point three correspond lines 1012-13: infinity results also if at some point one

⁷ In his extant remarks Alexander does not mention Epicurus by name. It is clear, however, from the context that his objections to those believing in the fourth proof of infinity in Aristotle are directed against the atomists and, more specifically, probably against the atomism of Epicurus. First, one of his sentences is in wording very similar to that of Epicurus in the same context. Alexander writes with respect to finiteness: ϵl $\mu \dot{\epsilon} \nu$ $\sigma \dot{\nu} \nu$ $\tau \dot{\rho}$ $\sigma \epsilon \mu \dot{\rho} \nu \dot{\rho} \nu$ $\tau \dot{\rho}$ $\epsilon \nu \dot{\rho} \nu \dot{\rho} \nu$ $\epsilon \nu \dot{\rho} \nu \dot{\rho} \nu$ $\epsilon \nu \dot{\rho} \nu$ $\epsilon \nu \dot{\rho} \nu \dot{\rho} \nu$ $\epsilon \nu \dot{\rho} \nu$

 8 It is not clear from the Greek of Simplicius whether $d\rho\chi\alpha\iota\sigma\tau\epsilon\rho\sigma\upsilon$ should go with the Epicureans (sc. $d\rho\chi\alpha\iota\sigma\tau\epsilon\rho\sigma\upsilon$ $\tau\hat{\omega}\nu$ $\pi\epsilon\rho$ ì $E\pi(\kappa\sigma\upsilon\rho\sigma\upsilon)$ or with Aristotle himself. A comparison with Themistius, however, indicates that Simplicius meant that the proof was older than Epicurus. Themistius writes: $\kappa\alpha\iota\tau\sigma\iota$ $\tau\hat{\delta}\nu$ $\lambda\hat{\delta}\gamma\sigma\upsilon$ $\tau\hat{\sigma}\hat{\upsilon}\tau\sigma\upsilon$ (sc. the proof based on the fourth proof for infinity in Aristotle) $\eta\gamma\hat{\alpha}\pi\eta\sigma\epsilon\nu$ $\sigma\hat{\upsilon}\tau\omega$ $E\pi(\kappa\sigma\upsilon\rho\sigma)$, $E\pi(\kappa\sigma\upsilon\rho\sigma)$, $E\pi(\kappa\sigma\upsilon\rho\sigma)$ $E\pi(\kappa\sigma\upsilon\rho\sigma)$ $E\pi(\kappa\sigma\upsilon\rho\sigma)$ $E\pi(\kappa\sigma\upsilon\rho\sigma)$ $E\pi(\kappa\sigma\upsilon\rho\sigma)$ $E\pi(\kappa\sigma\upsilon\rho\sigma)$ $E\pi(\kappa\sigma\upsilon\rho\sigma)$ $E\pi(\kappa\sigma)$ $E\pi(\kappa\sigma$

quantum stops being bounded by another. The close correspondence of the two proofs is strong evidence that lines 1. 1008–13 have come down to us essentially uncorrupted and that the various attacks in the past on their soundness can now be disregarded once and for all. The new proof also helps to determine the meaning of *rerum summa* in 1008.

With respect to rerum summa scholars writing after the edition of Lachmann in 1850 have been split roughly evenly between the meaning 'sum total of matter' and the meaning 'universe'. 10 Now, Simplicius, after giving the proof of infinity, states explicitly that this proof was, according to Alexander of Aphrodisias, used by preference by the Epicureans to assert the infinity of $\tau \hat{o} \pi \hat{a} \nu$. It is reasonable to assume that when Lucretius used this proof in 1. 1008–13 he used it for the same purpose. This evidence from Simplicius confirms the observation of Schulz that the meaning 'universe' for rerum summa is required because of lines 1. 1012–13.11 In these lines Lucretius maintains inter alia that the infinity of rerum summa would also result if matter were finite and void infinite. If rerum summa had the meaning 'sum total of matter' then Lucretius would be proving its infinity here by a proof which assumes that matter is finite! This absurdity disappears if rerum summa means 'universe'. 12 Then in lines 1012–13 Lucretius is found stating that the universe is infinite even if only one of its two components, void or matter, is infinite. To sum up, lines 1012–13 in combination with the statement in Simplicius are conclusive evidence for the meaning 'universe' of rerum summa in 1. 1008.

As I mentioned before, the striking similarity between the Epicurean proof for infinity in Simplicius with 1. 1008–13 is a guarantee for the soundness of the text of Lucretius here. In the past attempts had been made to condemn lines 1012–13 as an interpolation, to rewrite line 1012, or to separate line 1012 from line 1013 by postulating a lacuna between them. ¹³ The Epicurean proof now supplies an additional argument against these suggestions.

10 Among the more important scholars, the meaning sum-total of matter was proposed by P. E. Goebel, Observationes Lucretianae criticae et exegeticae (Bonn, 1854), 4. The same interpretation was supported by Munro in his edition in 1864, by Bockemüller's edition of 1873, by F. Stuerenberg, 'De carminis lucretiani libro primo', Acta Societatis philologae lipsiensis 2. 2 (Leipzig, 1874), 412, by F. Neumann, De interpolationibus lucretianis (Halle, 1875), 13, by J. Woltjer, Lucretii philosophia cum fontibus comparata (Gröningen, 1877), 34, by J. van der Valk in his edition of Bk. I of Lucretius (Kampen, 1903), by J. Mussehl, De Lucretiani libri primi condicione et retractatione (Tempelhof, 1912), 77, by W. A. Merrill, 'Criticism of the Text of Lucretius with Suggestions for its Improvement', University of California Publications in Classical Philology 3 (1916), 12, by K. C. Reiley, Studies in the Philosophical Terminology of Lucretius and Cicero (New York, 1909), 117, by C. Pascal in his edition of Bk. I (Second edition in Rome, 1928), by C. Bailey in his commentary to Lucretius, and by G. Müller, Die Darstellung der Kinetik bei Lukrez (Berlin, 1959), 97.

Rerum summa in the meaning omne or 'the universe' was advocated already in 1832 by J. N. Madvig, 'De aliquot lacunis codicum Lucretii', Opuscula Academica (Hauniae, 1887), 254. He was followed by Lachmann in the commentary of his edition in 1850, by T. Bindseil, above n. 2, p. 7, by F. Susemihl, 'Neue Bemerkungen zum ersten Buche des Lucretius', Philologus 44 (1885), 81, by Giussani in his edition of Lucretius (1896), by A. Brieger, 'Epicurs Lehre vom Raum...', Philologus N.F. 14 (1901), 529, by Smith and Leonard in their edition of Lucretius (1942), by P. R. Schulz, 'Das Verständnis des Raumes bei Lukrez', Tijdschrift voor Philosophie 20 (1958), 48, and by A. Barigazzi in his edition of selected passages of Lucretius (Turin, 1974).

¹¹ Above, n. 10, p. 50.

¹² The exact meaning of *rerum summa* in 1. 1008 cannot be established on the grounds of Lucretian usage alone because *rerum summa* in Lucretius can refer both to universe and to matter. For examples see Bailey's commentary to 1. 235.

¹³ F. Polle, 'Zu Lucretius', *Jahrbücher für classische Philologie* 93 (1866), 757–60 considered lines 1012–13 as an interpolation of a learned reader, partly because of his misunderstanding of the content of these lines but partly also because he argued that the elision in *alterum eorum*

Another neglected parallel to an Epicurean proof for the infinity of the universe may be found in Aristotle's Physics 206 b 33-207 a 2: Συμβαίνει δὲ τοὖναντίον ἄπειρον είναι η ώς λέγουσιν οὐ γὰρ οῦ μηδὲν ἔξω, ἀλλ' οῦ ἀεί τι ἔξω ἐστί, τοῦτο ἄπειρόν $\epsilon \sigma \tau \iota$. Neither Aristotle nor his commentators identifies the subject of $\lambda \epsilon \gamma \sigma \nu \sigma \iota$. However, both Epicurus and Lucretius argue for the infinity of the universe on the grounds that there is nothing outside of the universe to limit it. This is the comparable proof of Epicurus: 'Αλλὰ μὴν καὶ τὸ πᾶν ἄπειρόν ἐστι· τὸ γὰρ πεπερασμένον ἄκρον έχει· τὸ δὲ ἄκρον παρ' ἔτερόν τι θεωρεῖται ⟨ἀλλὰ μὴν τὸ πᾶν οὐ παρ' ἔτερόν τι θεωρείται· (Usener's addition) ωστε οὐκ ἔχον ἄκρον πέρας οὐκ ἔχει· πέρας δὲ οὐκ ἔχον ἄπειρον ἂν εἴη καὶ οὐ πεπερασμένον (Ad Herod. 41). The same proof is found in Cicero (De Div. 2. 103) and in Lucretius (1. 958-64) and is unmistakably referred to in Alexander of Aphrodisias.¹⁴ All these proofs rely on the assumption that whatever is not externally limited by something else is infinite. In the atomist physics only atoms and void exist. Since nothing else exists, there is nothing to limit them externally. Therefore, the universe, consisting as it does of void and atoms, must be unlimited.

This proof clearly relies on the implicit assumption that everything finite must be externally limited by something else. Different views were not unknown. It could, for instance, be assumed that something finite need not necessarily be bounded by something external to it. Now, the assumption that finite objects must be externally limited also was, as mentioned above, the essential point in the proof for the infinity of the universe attributed to the Epicureans by Simplicius and some other commentators. Therefore, one may with justification consider the $o \bar{b} \mu \eta \delta \dot{e} \nu \xi \xi \omega$ proof of infinity and the proof in Simplicius, found in Lucretius in 1. 1008–13, as parts of the same proof.

The οὖ μηδὲν ἔξω proof of infinity (a proof derived from the absence of an external limit) may go back to the fifth century B.C. Aristotle gives a proof for the infinity of the universe which has been attributed by most scholars to the Eleatic Melissus. ¹⁶ According to Aristotle ἕν καὶ ἀκίνητον τὸ πᾶν εἶναί φασι καὶ ἄπειρον ἔνιοι· τὸ γὰρ πέρας περαίνειν ἂν πρὸς τὸ κενόν (De gen. et corr. 325 a 14–16). The context in

in 1012 was against Lucretian practice. This latter argument enjoyed some popularity, being accepted by Stuerenberg (above, n. 10, p. 415) and by Neumann (above, n. 10, p. 13 n. 1). Line 1012 was attacked also by Woltjer (above, n. 10) who, not familiar with this proof of infinity, suggested non quoniam for aut etiam, thus reversing the required meaning. Finally, there was the well-known attempt by Lachmann in his commentary to place the lacuna not after line 1013 but rather after line 1012.

- 14 Above, n. 3, 104. 21–1: εἰ μὲν οὖν τῷ πεπερασμένῳ τῷ εἶναι πεπερασμένῳ ἐστὶν τῷ $\theta \epsilon \omega \rho \epsilon \hat{\imath} \sigma \theta a$ παρ' ἄλλο... Alexander seems to be using here the very language of Epicurus, who in the same context writes: τὸ δὲ ἄκρον παρ' ἔτερόν τι $\theta \epsilon \omega \rho \epsilon \hat{\imath} \tau a$ (Ad Herod. 41).
- 15 Aristotle assumed that what was whole had to be finite: $(\tau \delta \delta \lambda \nu sc. \dot{\epsilon} \sigma \tau l)$ οδ μηδέν $\dot{\epsilon} \delta \sigma \iota \nu$ εξω (*Physics* 207 a l) and understood the universe of Parmenides to be finite: $\tau \delta \delta \lambda \nu \nu$ (sc. of Parmenides) $\pi \epsilon \pi \epsilon \rho \dot{\alpha} \nu \theta a \iota$ (*Physics* 207 a 16–17).
- 16 Belief in the authorship of Melissus has been nearly unanimous. The only closely reasoned argument against his authorship seems to be that of J. Barnes, The Presocratic Philosophers I (London, 1979), 201. In his strongest objection he maintains that if the proof of the infinity of $\tau \delta \pi \hat{a} \nu$ found in Aristotle $-\tau \delta \gamma \hat{a} \rho \pi \epsilon \rho a (\nu \epsilon \nu) \hat{a} \nu \pi \rho \delta s \tau \delta \kappa \epsilon \nu \delta \nu$ were admitted among the other proofs of Melissus, it would introduce circularity. However, circularity would appear only if for this proof to be valid it were necessary, as is assumed by Barnes, to prove first that the universe $(\tau \delta \pi \hat{a} \nu)$ is one rather than many. It seems to me that this latter proof is not indispensable. There is no reason to assume that the concept $\tau \delta \pi \hat{a} \nu$ cannot here include everything that exists. In other words, even if what exists were many and not one, the many could be thought to be subsumed under $\tau \delta \pi \hat{a} \nu$ and be parts of its total being and extension. The proof that $\tau \delta \pi \hat{a} \nu$ is one would be necessary only if the many were thought not to be part of $\tau \delta \pi \hat{a} \nu$ and, therefore, were able to function as a limit to it.

Aristotle of this passage makes it clear that he is here referring to the Eleatics; the attribution of infinity to $\tau \delta \pi \hat{a} \nu$ points to Melissus in particular. This proof is in essence identical with the proof for the infinity of the universe found in Epicurus (Ad Herod. 41) and Lucretius 1. 958–64. Both proofs implicitly assume that if the universe were finite there would be something outside of it to limit it. Both proofs then assert that there exists nothing that could serve as a limit. In the atomist system only void and matter exist and they cannot form a limit of the universe, being themselves part of it. In the Melissean proof the void appears to be imagined as the only possible limit to the universe; but for the Eleatics the void did not exist. Therefore, the universe had to be infinite. As mentioned before, the Melissean proof, the proof for infinity attributed to the Epicureans in the commentators to Aristotle, and the Epicurean-Lucretian proof based on the absence of an external limit all depend on the same fundamental assumption, an assumption most concisely expressed in Aristotle: $\tau \delta \pi \epsilon \pi \epsilon \rho a \sigma \mu \epsilon \nu \delta \tau$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \epsilon \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu \delta \nu$ $\tau \epsilon \rho a \sigma \mu \delta \nu$

Because of their presence in earlier philosophers and because of the explicit statements in Simplicius and Themistius¹⁷ the other clauses of the limit-proof also precede Epicurus. Although the fundamental assumption of the proof may be as old as Melissus and may, for all we know, have been first enunciated by him, the clauses involving the endless succession of finite units are very unlikely to be Eleatic. Since for the Eleatics the existing was one, they would hardly have been interested in excogitating a demonstration which involved the derivation of the infinity of the universe from the infinite succession of two or more primary existents. In fact, Melissus explicitly denies that the infinite can be anything but a unity $(\tau \delta \tilde{\epsilon} \nu)$. ¹⁸ On the other hand, of all the Presocratic systems that of the atomists would find it most suitable and appropriate for its physics. Therefore, it seems reasonable to assume that even if the fundamental assumption of the limit-proof may have been made by Melissus, the clauses applicable to an infinite universe consisting of more than one infinite primary substance were added by a pre-Epicurean atomist, perhaps Leucippus. As is well known, similarities exist between the remains of Leucippus and Melissus. Scholarly opinion is about evenly split on the question of priority. If there was borrowing, it is also possible that the entire limit-proof was invented by Leucippus and that Melissus used those points of it which suited the Eleatic system.

That Leucippus was involved in formulating at least a part of the limit-proof may also be indicated by the proof in 1. 1008–1013 in Lucretius. The proof for infinity in lines 1012–1013 (infinity also results if finite quanta are succeeded by an infinite one) actually does not fit Epicurean physics. In it neither void nor matter can be finite even if the other component were allowed to be unbounded, therefore infinite. This is stated explicitly by Epicurus: Καὶ μὴν τῷ πλήθει τῶν σωμάτων ἄπειρόν ἐστι τὸ πῶν καὶ τῷ μεγέθει τοῦ κενοῦ· εἴ τε γὰρ ἦν τὸ κενὸν ἄπειρον, τὰ δὲ σώματα ὡρισμένα, οὐθαμοῦ αν ἔμενε τὰ σώματα, ἀλλὶ ἐφέρετο κατὰ τὸ ἄπειρον κενὸν διεσπαρμένα... εἴ τε τὸ κενὸν ἦν ὡρισμένον, οὖκ αν εἶχε τὰ ἄπειρα σώματα ὅπου ἐνέστη (Ad Herod. 42). Why would Lucretius, then, have inserted it in one of his proofs (lines 1012–13)? Perhaps Lucretius, or his source, felt that it would do no harm to demonstrate that the infinity of the universe could be established even by a proof weaker than the one derivable from the actual system of Epicurus. Also, it is possible that the original proof

¹⁷ For these statements in Simplicius and Themistius see n. 8.

¹⁸ For instance, Simplicius states of Melissus: ἀπὸ δὲ τοῦ ἀπείρου τὸ ἔν συνελογίσατο ἐκ τοῦ "εἰ μὴ ἔν εἴη περανεῖ πρὸς ἄλλο" (above, n. 5, 110. 5–6 or Diels–Kranz 1, 30 B 5).

was developed for a system in which one could envisage both an infinite alternation of finite quanta and the succession of the finite by the infinite. Areas of pure void bereft of matter are attributed for Leucippus by Diogenes Laertius: $\phi \epsilon \rho \epsilon \sigma \theta a \iota \dots \pi o \lambda \lambda \dot{a}$ σώματα παντοία τοίς σχήμασιν είς μέγα κενόν... 19 Large areas of pure void are deprecated by Epicurus, however: ὁ τοιοῦτος δύναται κόσμος γίνεσθαι καὶ ἐν κόσμω καὶ μετακοσμίω... ἐν πολυκένω τόπω καὶ οὐκ ἐν μεγάλω είλικρινεῖ καὶ κενώ $\kappa \alpha \theta \acute{\alpha} \pi \epsilon \rho \dots \phi \alpha \sigma \iota \nu \dots (Ep. ad Pyth. 89)$. Here Epicurus clearly is taking a stand against an earlier view, a view attested for Leucippus, which envisaged areas of pure void. Perhaps point three of the commentators' proof of infinity, found also in line 1012–13 of Lucretius, was devised by Leucippus. Although there is no information, it may be that he thought that not only the entire void, but even large sections of it, could be infinite in extent. The concept of multiple infinities is well attested in atomism. Atoms are infinite in number and in extension, 20 yet they fit into an infinite void with room left over for their motions. In earlier atomism the shapes of atoms were infinite. The number of atoms of each shape is infinite, too. Moreover, the number of kosmoi is infinite. Wherever in the universe one takes his stand it continues on to infinity in all directions (Lucret. 1. 965-67). No matter how much of it the lightning traverses, its journey remains infinite (Lucret. 1. 1005). It is, therefore, possible that Leucippus could have assumed that parts of the void could be infinite, too. In fact, such a conclusion would have been quite logical. If atoms were thought to be infinite in extent, then the void occupied by them would have to be infinite, too. However, if they could move, the extension of the void had to surpass that of the atoms. In turn, the infinite number of atoms of each shape, although infinite in extension, could be assumed to occupy only a small part of the void occupied by all the atoms. In consequence, Leucippus may have thought that in some parts of the universe infinity would be arrived at by the endless alternations of finite extensions of void and matter, in other parts by the successions of finite void or matter by a section of infinite void or infinite matter.21 As I mentioned above, the antiquated section of the proof may have been kept by the later atomists because it demonstrated the infinity of the universe even if either matter or void were assumed to be finite. The universe would then be a fortiori infinite if both of its parts, the void and the atoms, were shown to be infinite. However, it was now necessary to point out that this part of the limit-proof did not apply to contemporary atomism. This correction is found in Epicurus (Ad Herod. 42), whose denial of limited void or matter must no doubt be placed in the Lucretian lacuna after 1. 1013. Perhaps this denial is referred to by Themistius, who taxed Epicurus with having appropriated the limit-proof of infinity and with having supplied it with μικραῖς τισι καὶ φαύλαις προσθήκαις. 22 Themistius does not tell us what these $\pi \rho o \sigma \theta \hat{\eta} \kappa a \iota$ were but they may have been the alterations to the original form of the limit-proof of infinity.

Although it seems to me that the specifics of their system point to atomists as the originators of the limit-proof of the infinity of the universe, with Melissus being a

¹⁹ D-K II, 67 A 1, 28-30. The μέγα κενόν is also attested by Hippolytus 67 A 10, 25-6, where Roeper plausibly substituted it for the μετάκοινον of the mss.

²⁰ Above, n. 1.

²¹ Leucippus, like Democritus and Epicurus after him, appears to have held that atoms in juxtaposition never formed a true unity, i.e. they were separated by some void. For discussion and references see W. K. C. Guthrie, A History of Greek Philosophy II (Cambridge, 1965), 390. If so, for Leucippus a section of infinite matter would have to be a single atom of infinite extension. The notion of such an atom may be encouraged by the much-debated cosmos-sized atoms envisaged, according to Aetius, by Democritus (DK II, 68A47). Epicurus, however, held that atoms were too small to be seen (Ad Herod. 55-6).

²² Above, n. 4, 100. 8-10.

possible pretender, the possibility of Pythagorean background cannot be excluded. Simplicius, on the authority of Eudemus, reports the proof for infinity of the Pythagorean Archytas, ²³ who was active in the first half of the fourth century B.C. A form of this proof is also used by Lucretius in 1. 968–84. In Simplicius and other commentators it is given as an illustration of the fifth proof of infinity found in Aristotle: $\delta i \dot{\alpha} \gamma \dot{\alpha} \rho \tau \dot{\alpha} \dot{\epsilon} \nu \tau \dot{\eta} \nu o \dot{\eta} \sigma \epsilon \iota \mu \dot{\eta} \dot{\nu} \pi o \lambda \epsilon (\pi \epsilon \iota \nu \ldots \delta o \kappa \epsilon i (\pi \epsilon \iota \nu \cos \epsilon i \nu a \iota \ldots \kappa a) \tau \dot{\epsilon} \dot{\epsilon} \omega \tau o \hat{\nu} o \dot{\nu} \rho a \nu o \hat{\nu} (Physics 203 b 23–5)$. Furley²⁴ and Cherniss²⁵ notice the similarity of the fifth proof in Aristotle to his fourth proof, the limit proof, with Cherniss suggesting that the Pythagorean Archytas, may have been its originator. This possibility cannot be excluded, especially if we agree with Raven that Archytas, himself a contemporary of Plato, may be giving here an illustration of an earlier Pythagoran doctrine. ²⁶

It is probably futile to attempt to search for a single originator of the fundamental idea of the limit-proof, the intuitively obvious notion that whatever is finite must have something beyond it. However, the atomists may very well be responsible for its more elaborate versions, as illustrated by Epicurus, Lucretius, Aristotle, and his commentators. After all, the commentators attribute them to Epicurus by name, most of the known borrowings of Epicurus are from the earlier atomists, and the limit proof of the universe as a whole seems to fit into the atomist system best.

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^{23 &#}x27;Αρχύτας δέ, ὡς φησιν Εὔδημος, οὕτως ἠρώτα τὸν λόγον ἐν τῷ ἐσχάτῳ οἶον τῷ ἀπλανεῖ οὐρανῷ γενόμενος, πότερον ἐκτείναιμι ἄν τὴν χεῖρα ἢ τὴν ῥάβδον εἰς τὸ ἔξω ἢ οὕ; καὶ τὸ μεν οὖν μὴ ἐκτείνειν ἄτοπον εἰ δὲ ἐκτείνω, ἤτοι σῶμα ἢ τόπος τὸ ἐκτὸς ἔσται. διοίσει δὲ οὐδὲν ὡς μαθησόμεθα. ἀεὶ οὖν βαδιεῖται τὸν αὐτὸν τρόπον ἐπὶ τὸ ἀεὶ λαμβανόμενον πέρας, καὶ ταὐτὸν ἐρωτήσει, καὶ εἰ ἀεὶ ἔτερον ἔσται ἐφ' δ ἡ ῥάβδος, δῆλον ὅτι καὶ ἄπειρον (above, n. 5, 467. 26–32).

²⁴ David J. Furley, 'Aristotle and the Atomists on Infinity', in I. Düring (ed.), *Naturphilosophie bei Aristoteles und Theophrast* (Heidelberg, 1969), 92–3.

²⁵ Harold Cherniss, Aristotle's Criticism of Presocratic Philosophy (New York, 1964, Repr. of 1935 ed.), 20-1.

²⁶ J. E. Raven, Pythagoreans and Eleatics (Cambridge, 1948), 80-1.